SIMULATION OF PATTERNS ON CIRCULAR KNITTED TUBES

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This method paper explores the creation of a pattern with parallel curves on a machine knitted tube in the frame of an artistic research project. Using modular arithmetic and the software *Processing*, the initially empirically generated pattern is analyzed and simulated. The results are then used in order to create intentional iterations of the parallel curve pattern, determining the winding of the yarn for dyeing and the lengths of the yarn strands. The paper draws a connection from a craft based technique (knitting) to mathematics.

1. HISTORY AND CONTEXT

This project took its starting point within the context of Huijnen's research and art practice that investigates the mouth, speech and affect from a physiological, historical and feminist perspective. Her goal to create a knitted tube evoking aspects of the human mouth and esophagus (gullet) would lead to a deeper investigation of circular knitting, yarn dyeing and mathematical principles.

For this project Huijnen had access to an antique Creelman Brothers *Money-Maker* circular sock knitting machine. The Creelman Brothers had founded their knitting machine company in 1876 in Georgetown, Ontario (Canada) and in 1893 the first *Money-Maker*, one of several other circular knitting machines designed for use in the home, came to the market. Those knitting machines are a manifestation of the early years of the industrialization process and of factory mechanization. (Terry 2010)

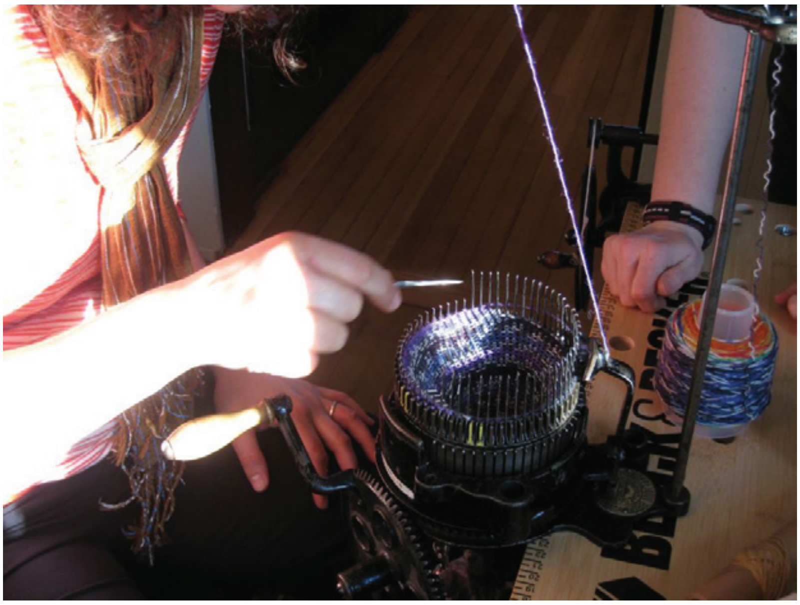


Fig. 1 Working with the *Money-Maker* sock knitting machine.

Huijnen's motivations for using this knitting machine were twofold. It provided a technical benefit because it worked fast and produced regular stitches (working the machine still required concentration, precise counting and awareness for its failures). The second motivation was the fascination for a machine doing a 'typical' female work and this way embodying two conventionally differently gendered work spheres. While writing this paper the authors had realized that the questionable existence of gendered work spheres had partly reached within their own collaborative process. Excluding the artistic conception of the work, Huijnen realized the knitting related part of the project and Wanner the mathematical part.

2. ARTISTIC GOALS

The goal of Huijnen's art project lied in creating a pattern on a knitted tube that would visualize the peristaltic[1] movement of food through the esophagus. Looking for a way to reference this process of muscular contractions, schematic representations of the peristaltic movement served as visual reference.

They abstracted the fleshy and slippery aspect of the human gullet, transformed it into a drawn tube with waved outlines and a limited colour palette. For the knitted tube a similar level of abstraction should be obtained. Without using a descriptive colour palette, the reminiscence of the gullet should function on a symbolic level through a periodic, cyclic or wave pattern and a reduced use of colour: a dark stain on naturally white sheep yarn.

In addition, Huijnen wanted the pattern to emerge from the ball of yarn itself, instead of using two differently colored yarns to create a predefined pattern. An immediate correspondence between the original ball and the resulting tube could conceptually stand for the transformative process of digestion. Also when looking closely at the knitting movement of the sock machine needles, one can observe how the yarn is continuously grasped by the needles and pulled into a tube. One can then imagine why the opening and closing latches of the needles are called 'tongues'. (Iyer, Mammel and Schäch 1991, 54)

In order to create patterns that would be intrinsic to the ball of yarn, Huijnen decided to dye the yarn in a straightforward way: one half of a yarn ball was dipped in hair dye as schematized in figure 3. Then the yarn was dried and machine knitted into a tube.

The dyed yarn strand can be expected to consist of dashed lines with increasing length, as illustrated in figure 4.

Fig. 2 Peristalsis (schematic).

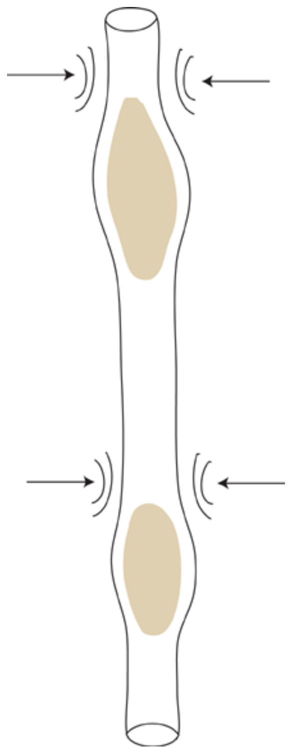
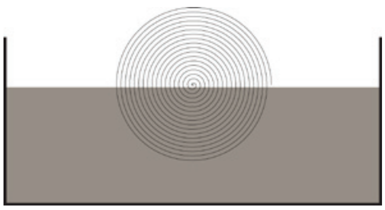


Fig. 3 Dyeing of yarn ball (schematic).



1 The peristaltic movement or peristalsis is a muscular contraction that proceeds in waves and pushes the bolus of food towards the stomach.

Fig. 4 The presumed pattern on the yarn.

The knitted pattern emerging from this process and its reiterations will be presented in the following parts of this paper. The focus will be on the calculation and comprehension of the empirically generated patterns. The artistic aspect of the project will not be discussed in detail.

3. PATTERNS PRODUCED

The first generated pattern exposed a color gradient from white to brown between the two ends of the tube as shown in figure 6. The yarn ball placed in front of the tube in figure 6 visually demonstrates to the art viewer how the yarn ball was stained. Analyzing the pattern, it became clear, that only the outer strands of the yarn ball had been colored, as it is illustrated in figure 5.

In a second iteration, another yarn ball was dipped in hair dye, with an effort to stain the entire half. The dyeing of the yarn is both influenced by the fluidity of the dye and the absorbing quality of the yarn. When dyeing yarn while it's wet, the color diffuses further along the yarn strands which explains the varying color intensities of the patterns that follow.

Fig. 5 Dye not fully penetrating into yarn ball, thus leaving a large inner section of the yarn entirely white (schematic).

Fig. 6 *Tube and Yarn Ball*, 2012. First generated tube with stained ball of yarn **Fig. 7** Second tube

The result of the second tube (figure 7) surprised. Instead of having a more pronounced color gradient from white to brown, another phenomenon had occurred: a delicately drawn pattern of 4 and further up 3 parallel curves (some of which on the invisible part on the back side of the tube) could be distinguished. How had this pattern been created?

4. A MATHEMATICAL MODEL OF KNITTING IN THE ROUND

Discussing this unexpected phenomenon between the authors, it was decided to use the visualization software *Processing* to simulate the generative process leading to these patterns. The authors were curious, whether the patterns could be reproduced in this simulation, and if it would be capable of making predictions and being used as a design tool to anticipate further iterations.

The following sections will introduce the mathematics and simulation assumptions step by step.

4.1 MATHEMATICS AND KNITTING

In her book *Häkeln + Stricken für Geeks (Crocheting and Knitting for Geeks)*, DIY researcher Verena Kuni describes parallels and connections between mathematics, computation and knitting. Kuni states that counting and numbers decide on the execution of the knitwear.

> *Increasing, decreasing, colored and other patterns – all this is applied mathematics with a computational reference; a crocheting or knitting instruction can rightly be considered an algorithm. (Zunehmen, abnehmen, Farb- und andere Muster – alles das ist angewandte Mathematik mit informatischem Bezug; eine Häkel- oder Strickanweisung kann mit Fug und Recht als Algorithmus bezeichnet warden.) (Kuni 2013, 9-10)*

In her research, mathematician and textile artist Ellen Harlizius-Klück reveals a similar and more ancient relationship between mathematics and craft. She describes weaving since Greek Antiquity in relation to dyadic arithmetic, the arithmetic of odd and even numbers. (Harlizius-Klück 2008, 2) Stating that all weaving is done in dyadic terms, the only choice being between “zero (warp-thread down) and one (warp-thread up)” (Harlizius-Klück 2008, 5-6) she accordingly relates the history of weaving to the origins of computing and the fitting of a pattern into a woven fabric to mathematic calculations of numeric divisibility. In analogy to weaving, knitting could also be described as binary, with a choice between a purl and a knit stitch.

As far as knitting in the round is concerned, Verena Kuni introduces modular arithmetic (Kuni 2013, 9-10), a field of mathematics suited to describe repetitive patterns on objects knitted in the round – usually socks – and refers to the knitting projects of mathematician Sarah-Marie Belcastro. Belcastro's pattern of striped *Sublimation Socks* for example is based on a sequence of integer numbers. (Belcastro 2012) In a process between arithmetic and trial-and-error, the pattern is adapted to the shape and design of the socks. An equation for the calculation of the sock rows is also provided.[2]

Huijnen's pattern, empirically discovered, directly relates to these mathematical sock knitting patterns and modular arithmetic.

4.2 MODELING A PATTERN WITH VERTICAL STRIPES

The further investigation will rely on the book *Making Mathematics with Needlework* by Belcastro and her colleague Carolyn Yackel. Belcastro and Yackel present modular arithmetic as an opportunity for a designer to "create a pattern that looks complex but is simple to execute." (Belcastro and Yackel 2007, 95)

They introduce two essential modular dimensions: the row length l_r – the length of yarn needed to complete one row – as well as the pattern length l_p – the length of yarn needed until the pattern repeats itself. If the pattern is supposed to repeat itself on every row, the row length l_r needs to be an integer multiple n of the pattern length (93), so that a resonance (or matching) occurs between the two modular dimensions.

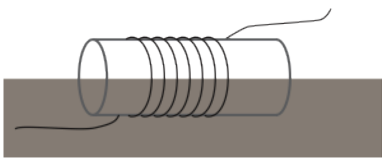
$$l_r = n \cdot l_p \ (n \in Z)$$

In our case, the circumference of the knitted tube was measured to be 70 cm.[3] To verify our hypothesis, that the pattern will repeat itself in every row, we chose a cylindrical winding to result in stained yarn bits of a pattern dividing the row length l_r = 70 cm. To obtain n = 4 vertical stripes, a pattern length l_p = 17.5 cm has to be generated, by winding yarn around a cylinder of d = 5.6 cm diameter, as figure 8 illustrates.

$$l_r = 4 \cdot l_p = 4 \cdot \pi \cdot d$$

Figures 9 (computer simulation) and 10 (photo of knitted tube) show how this pattern was confirmed in

Fig. 6 Cylindrical winding and dyeing



2 On the crafters forum *Ravelry*, her pattern as well as the mathematical equation behind the socks can be downloaded *http://www.ravelry.com/designers/sarah-marie-belcastro*

3 Corresponding to a row length of 60 stitches this results in a conversion factor of about 1.17 cm per stitch. Note that due to the flexibility and softness of the yarn material, these measures are approximate and slight variations are to be expected.

a simulation and verified by actually knitting it. The knitted version shows roughly vertical lines with some variations. The authors attribute the zigzag variations to varying tightness and the overlapping of thread when winding the yarn around the cylinder. This may provide an account of the accuracy limits of the method.

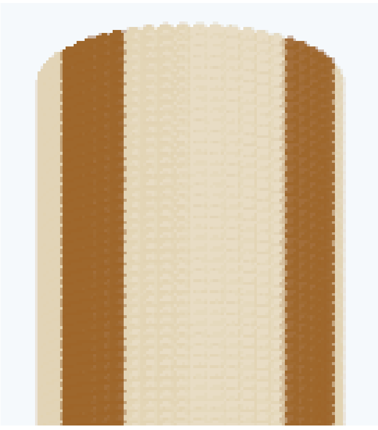


Fig. 9 Pattern with straight lines, the trivial case of the computer simulation **Fig. 10** Pattern with straight lines, some variations occur

4.3. MODELING A PATTERN WITH DIAGONAL STRIPES

Belcastro and Yackel then suggest a way of producing a pattern of diagonal stripes based on a simple counting algorithm:[4]

> *Suppose we wanted to have diagonal stripes advancing across the sock […]. We could achieve this by having a pattern length of 61, with a pattern consisting of a block of navy followed by a block of white. The shift creating the advance arises because 61 ≡ 1 mod 60. […] To have the diagonal going the other direction, use a pattern length of 59.* (95)

Based on the insight that "when l_p does not divide l_r, the pattern does not 'line up' from one row to the next" (95), Huijnen chose a pattern length of 18 cm, resulting from winding yarn around a cylinder of diameter 5.75 cm. With these parameters, a pattern of 4 diagonal stripes was expected, slightly shifted by an offset of 2 cm in each row (or by about 5 stitches every 3 rows).[5]

$$(l_p \cdot n) \equiv offset \bmod l_r$$

or with numbers:

$$(18\ cm \cdot 4) \equiv 2\ cm \bmod 70\,cm$$

4 Belcastro and Yackel's pattern was to be obtained with yarns of two different colors: "In order to make a pattern using colors, at some point more than one color of yarn must be used when making the loops." (92)

5 Modular arithmetic was developed for integer numbers, and works well with stitches – a countable entity that is represented well by integers. Our method however presents patterns of a continuous length range, and is not limited to integers. The authors bend the mathematics a bit here, but their point can be made with both integer and floating-point numeric values.

Figures 11 and 12 show how this pattern was confirmed in a simulation and verified in the knitting machine. The verification with the knitting machine shows variations in the slope of the diagonal stripes, which the authors attribute to the same accuracy limits mentioned earlier.

Fig. 11 Pattern with diagonal lines, obtained by offsetting pattern- and row lengths. (simulation) **Fig. 12** Pattern with diagonal lines, photograph of tube

4.4 MODELING A PATTERN WITH VARYING LINE LENGTHS

With this work done, we can proceed to our main modeling situation: the attempt to generate a periodic or cyclic pattern based on dyeing half a ball of yarn. In the model, the yarn ball is represented with a flat Archimedean spiral.[6] The ball of yarn is immersed halfway as illustrated in figure 3, so that subsequent white and stained bits of yarn have the same length. The color (white or stained) at a specific point on the circular tube will be a function of:

- the specific point x along the yarn
- the pattern with varying pattern lengths $l_p(x)$ along the yarn
- the row length l_r of the tube

Modular arithmetic is not directly applicable in this case, because the pattern length is not constant, but will increase from beginning to the end of the yarn, as figure 4 illustrates it. However, a resonance – an area with locally matching patterns – can be predicted to occur around areas x_0 of the thread, where the row length l_r is an integer multiple of the pattern length l_p.

$$x \sim = x_0 \mid (l_p(x_0) \cdot n) \equiv 0 \bmod l_r$$

With a row length of 70 cm, we expect this resonance to occur for pattern lengths l_p of 10 cm (n = 7), 11.7 cm (n

6 The spatial dimension of the spherical ball of yarn is neglected with this assumption. While this may seem a drastic abstraction, it will qualitatively predict a tube based on a dashed thread, with increasing dash lengths.

= 6), 14 cm (n = 5) etc. Our model does not have enough quantitative prediction power to predict the exact points x along the thread. But pattern lengths along the yarn will be between 0 and d · π (d is the diameter of the yarn ball).

Both the computer model based on the assumption of the Archimedean spiral, as well as the resulting tube confirm these conclusions: figures 13 and 14 show a tube with the parallel curve patterns around specific points, as well as dense almost horizontal lines in between. In both the simulation and the photograph of the actual knitted tube, the density of curve patterns increases from top to bottom. This corresponds to the knitting direction from the outer windings of yarn to the inner ones, in which the pattern length of the stained stretches decreases. In between these decisive resonance patterns, the parallel curves become almost horizontal lines, before they approach a point of resonance again.

Fig. 13 Computer simulation **Fig. 14** Knitted tube with several parallel curve patterns

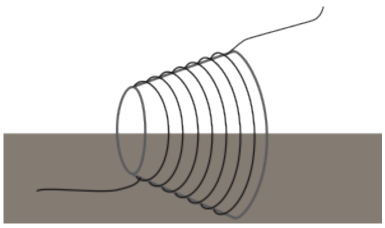
4.5 TOWARDS THE DESIRED PATTERN

The pattern obtained with a ball of yarn shows a progression from the outside to the inside of the yarn ball: there is an increasing fragmentation of shorter and shorter lines (pattern lengths). To obtain a periodic pattern, it is advisable to select a range of pattern lengths yielding interesting patterns:

$$l_p(x_{min}) < l_r / n < l_p(x_{max})$$

Then several pieces of yarn have to be wound in this way, so they all obtain pattern lengths between these boundaries. Adding these yarns in series will then result in the desired pattern. Further research is needed to identify suited yarn winding methods. For now, we suggest to proceed with a winding around a truncated cone with a lower diameter $l_p(x_{min}) \div \pi$ and bigger diameter $l_p(x_{max}) \div \pi$.

Fig. 15 Winding around a truncated cone.



5. FURTHER PATTERNS AND OUTLOOK

In order to create a composed knitted tube where the parallel curve pattern reoccurs in a cyclic and customized manner further experimentation and research will be invested in a more consistent winding of the yarn through a mechanical yarn winder and the specific design of winding objects.

Fig. 16 Periodic tube, composed of 3 subsequent threads of yarn wound up according to the procedurein figure 15 around an area of resonance.

Still another phenomenon caught the authors' attention upon close observation of one of the knitted tubes. One could notice slight distensions within the outlines of the tube, as can be seen in figure 17. These can be due to knitting with yarn that is still slightly wet, taking breaks during the knitting process and thus overstretching the yarn while it is fitted onto the needles of the knitting machine. As other effects that occurred empirically during the process of this project, this effect promises in a further step to be systematically developable into a shape not unlike the desired peristaltic tube.

Fig. 17 Detail of knitted tube showing an overstretching

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